Consider an object outside of r_{ergo} w/ $E_0 > 0$. It enters the ergosphere and splits into

$$E_0 = E_1 + E_2 \quad \Rightarrow \quad E_1 > E_0 > 0 \quad \text{so } E_1 \text{ can leave the ergosphere, but the } E_2 \text{ is trapped.}$$
$$>0 \qquad >0 \qquad <0$$

To an outside observer an object w/ E_0 approaches the BH, delivers negative energy E_2 (or siphons off energy $-E_2$) and then leaves. Now the energy E_2 includes contributions from the rest mass and motion of the object, but once it is absorbed and included in the energy of the BH, as discussed earlier, it must now appear as a contribution to the mass of BH. In other words $E_2 = \delta M_{BH}$. But since $E_2 < 0$, we are reducing the mass of the BH.

Now to get this process to work, i.e. to have E_2 be absorbed and E_1 be on a trajectory that leaves the ergosphere, one can show that $J_2 \leq \frac{E_2}{\Omega_H}$ where $\Omega_H = \frac{a}{r_+^2 + a^2}$ is the angular velocity of the horizon. Since $E_2 < 0$ this tells us that J_2 and Ω_H have opposite signs, hence the absorbed object must have angular momentum opposite in direction of the BH. But this means that absorption of 2 reduces the angular momentum of the BH by

$$\delta J_{BH} = J_2.$$

One important thing to remember is that this process requires an ergosphere region. Therefore the limit of this is when $J_{BH} = 0$ or $Kerr \rightarrow Schwarzschild$ or $r_{ergo} \rightarrow r_+ = 2GM$ beyond which nothing escapes.

For a perfect Penrose process $J_2 = \frac{E_2}{\Omega_H}$ or $\delta J = \frac{\delta M}{\Omega_H}$ (changes for BH, henceforth just $\delta J, \delta M$)

So we start with M_{Kerr} and reduce it w/ Penrose process to $M_{Schwarzschild}$. Shouldn't the horizon area get smaller? Not so fast.

First of all, since $r_+ = GM + \sqrt{G^2 M^2 - a^2}$ where $J = Ma$. From this, since both M and J are decreasing, it isn't immediately obvious what is happening to r_+. However we really should be considering the horizon area which is nontrivially related to r_+ by the geometry.

$$A_H = \int \sqrt{\det \gamma} \, d\theta d\phi \qquad \gamma_{ij} - \text{induced metric on horizon from } ds^2 \text{ w/ } r = r_+, \ dr = dt = 0$$

$$= 4\pi (r_+^2 + a^2)$$
$$= 8\pi G^2 M^2 + 8\pi \sqrt{\underbrace{G^4 M^4 - G^2 M^2 a^2}_{J^2}}$$

If we vary δM and δJ we find:

$$\delta A_H = \underbrace{\frac{8\pi G a}{\Omega_H \sqrt{G^2 m^2 - a^2}}}_{>0} \underbrace{(\delta M - \Omega_{H} \delta J)}_{\text{recall that} \quad \delta J \leq \frac{\delta M}{\Omega_H}}$$

So $\delta A_H \geq 0$ and the area theorem is (nontrivially!) obeyed.

The absolute nondecreasing of A_H sounds a bit like entropy. In some sense, we should expect a BH to have some observable entropy (we can observe A_H) since:

Beckenstein: If a BH had no observable entropy we could take an external system w/ entropy S_0 and upon throwing it into the BH, decrease the entropy of the observable universe, thus violating the 2nd law of thermodynamics.

To preserve the 2nd law, BHs must admit an observable entropy. One might think to use the mass M of the BH to correlate w/ entropy, but the Penrose process allows M to decrease in certain cases. It is only A_H which is a suitable proxy for entropy

But this already tells us something deep. While entropy usually scales w/ the volume of a system, in this case it scales w/ the area. This points to the holographic nature of gravity, since information from 4D is captured in a 3D surface.

Quantifying this idea, from the Kerr case:

$$\delta M = \frac{K}{8\pi G}\, \delta A + \Omega_H\, \delta J \qquad \text{w/} \quad K = \frac{\sqrt{G^2 m^2 - a^2}}{2Gm(Gm + \sqrt{G^2 m^2 - a^2})}$$

K is the "surface gravity" of the BH, or roughly, how strong the gravitational pull is near the horizon.

Comparing to $dE = TdS - PdV$ we would associate:

$$E = M$$
$$-PdV = \Omega_H\, \delta J$$
$$TdS = \frac{K}{8\pi G}\, \delta A$$

It is tempting to identify $T = \frac{K}{8\pi G}$ and $dS = \delta A$, but in truth the split isn't obvious

Enter Hawking: Hawking considered QFT in the curved geometry, near the horizon of a BH. Note: He was not doing quantum gravity (which is still not completely understood), he was doing perfectly well-defined QFT w/ minimal coupling.
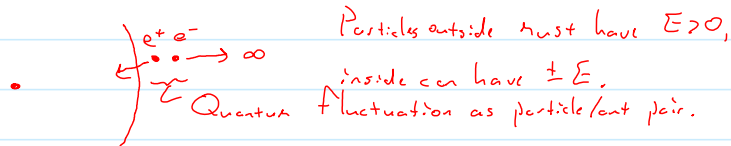
Even if you don't trust QFT in curved space, you could make do w/ QFT in flat space and then apply the equivalence principle.

The Unruh effect is the consequence of uniform acceleration in flat space. A uniformly accelerated observer in flat space experiences not $\mathbb{M}^4$, but rather a Rindler spacetime. Quantizing a field in terms of Rindler time is very different than w/ Minkowski time (we use time to identify allowable positive frequency modes from $e^{i\omega t}$ dependence). The end result is that a uniformly accelerated observer in the vacuum (no particles) of $\mathbb{M}^4$ actually sees a thermal distribution of all allowed particle types (dominated by lowest mass) coming at them w/ the temperature $T = \frac{a}{2\pi}$ where a is the acceleration of the observer

But now we use the equivalence principle to replace $a = K$ and find $T = \frac{K}{2\pi}$
and hence $dS = \frac{\delta A}{4G} \Rightarrow S = \frac{A}{4G}$

Black Hole Evaporation

Perhaps the most surprising aspect of Hawking's result is that BHs seem to radiate. The cartoon version of this is:

$e^+ e^- \longrightarrow \infty$

Particles outside must have $E > 0$, inside can have $\pm E$.
Quantum fluctuation as particle/ant pair.

Why surprising? Conservation of energy implies that the BH is losing energy and hence mass in this process and (as easily seen for Schwarzschild) this means the horizon area is decreasing. Does this violate the area theorem? Actually no! The area theorem assumed a weak energy condition ($\sim \rho \geq 0$), but these quantum fluctuations can have $\rho < 0$, hence violating the w.e.c.

So BHs can evaporate! Of course for large BHs this rate is small especially compared to any accretion. However for <u>microscopic BHs</u> it leads to them being very short-lived.
threatened LHC

Black Hole Information Paradox

Perhaps the most <u>perplexing</u> thing about BH evaporation is the seeming loss of information. First note that in principle, in an otherwise empty universe and given a long enough time, any BH will completely evaporate.
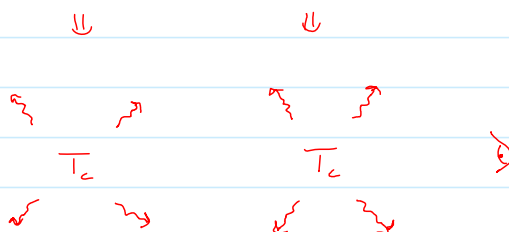Now consider 2 non-rotating, equal mass cows:

Male   Female

At this stage you could
that information is
"hidden" behind horizon.

$r = 2GM_c$ $r = 2GM_c$
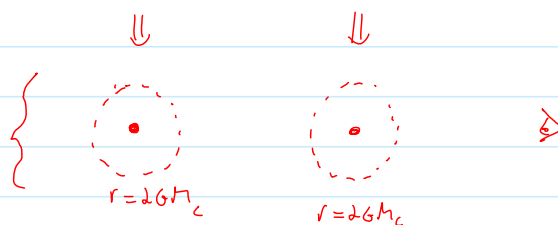
Uh-oh... no more horizon,
only T_c radiation which
is identical!

T_c T_c

Resolving this puzzle is almost certainly going to require a well-understood quantum theory of gravity!